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High-Frequency Sum-Rule Expansion of Relativistic Quasi-One-Dimensional Quantum Plasma Dielectric Tensor. II: Spin Effect

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A high-frequency sum rule for all elements of the relativistic quasi-one-dimensional quantum plasma with spins at $T = 0$ K are derived. It is found that the spin either enhances or reduces the frequency of oscillation, depending on the orientation of the spin to the external magnetic field.

1. INTRODUCTION

High-frequency sum-rule expansions of the full response tensor of nonrelativistic and relativistic quasi-one-dimensional quantum plasmas with spinless particles at $T = 0$ K are known (Genga, 1988*a*, 1992*a,b*). However, for quantum particles with spin the only result pertains to the nonrelativistic case (Genga, 1992*c*).

In this work I consider, as in the spinless particle situation (Genga, 1992*d*), the high-frequency behavior of the full dielectric tensor in an anisotropic system of relativistic quantum plasmas with spin particles at $T = 0$ K in the presence of an external magnetic field up to order ω^{-5}. I restrict my treatment to a situation where a strongly magnetized, strongly coupled electron plasma is being generated in a laboratory as in the Malmberg–O'Neil experiment. The radiation effect in such situations is negligible, unlike in an astrophysical plasma; the plasma density is assumed to be of the order of 10^{29} particles per unit volume. The Hamiltonian formalism is applied to derive the high-frequency sum rules as in the above-mentioned nonrelativistic case. Due to interaction it is known

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(Goldstone, 1957; Jancovici, 1962; Genga, 1992*a*,*b*,*d*) that an electron may jump from one state inside the Fermi sphere to an unoccupied state, thereby creating a hole known as a "Fermi hole." Further, the jump of an electron from a negative-energy state to an occupied positive-electron-energy state leads to the creation of a positron–electron pair with a positron as a hole known as a "Dirac hole." The interaction is therefore due to both spin and Coulomb interactions. Hence, the system is described by a set of unperturbed states, which allow for positrons and electrons.

I review the method of derivation below in this section. In Section 2 the general relations between the external or current–current response function sum-rule coefficients and those of the dielectric tensor are reviewed; further, the exact ω^{-2}, ω^{-3}, ω^{-4}, and ω^{-5} sum-rule coefficients for the transverse element are obtained. The long-wavelength limit of the results and the possible implications for the dispersion relation of the high-frequency plasma modes are considered in Sections 3 and 4, respectively.

The total electron current at point X_1 is given by

$$j(x_i) = \frac{e}{2} \sum [\mathbf{V}_i \delta(\mathbf{x} - \mathbf{x}_i) + \delta(\mathbf{x} - \mathbf{x}_i)\mathbf{V}_i] \tag{1}$$

where $\mathbf{V}_i$ is the group velocity of the free particle i with spin. The total energy for such a particle is given by (Johnson and Lipmann, 1949; Berestetskii *et al.*, 1978; Baym, 1974; Sakurai, 1987; Bjorken and Drell, 1964)

$$E = (\bar{\Pi}^2 c^2 + m^2 c^4 - 2ec\hbar \mathbf{B}^0 \cdot \mathbf{S})^{1/2} \tag{2}$$

where

$$\begin{aligned} \bar{\Pi} &= \mathbf{P} - \frac{e}{c}\mathbf{A}^0(\mathbf{r}) - \frac{e}{c}\mathbf{A}(r_1, t) \qquad \text{(generalized momentum)} \\ \mathbf{A}^0 &= \frac{1}{2}\mathbf{B}^0 \times \mathbf{r} \qquad \text{(external vector potential)} \end{aligned} \tag{3}$$

and $\mathbf{A}(\mathbf{r}_i)$ is self-consistent vector potential; the group velocity is given by (Genga, 1992*a*,*b*,*d*)

$$\mathbf{V}_i = \gamma^{-1} \frac{\bar{\Pi}_i}{m} \tag{4}$$

where

$$\gamma^{-1} = \left(1 + \frac{u^2}{c^2}\right)^{-1/2} \tag{5}$$

the relativistic factor with $\mathbf{u}$ defined as

$$\mathbf{u} = \left(\frac{\Pi^2 - 2e\hbar c^{-1}\mathbf{B}^0 \cdot \mathbf{S}}{m^2}\right)^{1/2} \tag{6}$$

Since I am only interested in the response function of the electron system, I first take the Fourier transform (1) followed by the expectation value of the resultant to obtain

$$\langle J_k^{\mu}(\omega)\rangle = e\langle j_k^{\mu}(\omega)\rangle - \frac{e^2 N}{mc}\gamma^{-1}T_{\mathbf{k}}^{\mu\nu}A_{\mathbf{k}}^{\nu}(\omega) \tag{7}$$

By applying perturbation theory (Pines and Nozières, 1966; Genga, 1988*a*,*b*, 1989, 1992*a*–*d*), it is found that

$$\langle j_{\mathbf{k}}^{\mu}(\omega)\rangle = -\frac{e^2}{c^2}\sum_{np}\omega^{-1}\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \times\left[\frac{1}{\omega-\omega_{n0}(\mathbf{p},\mathbf{p}+\hbar\mathbf{k}/2)+i\eta}-\frac{1}{\omega+\omega_{n0}(\mathbf{p},\mathbf{p}-\hbar\mathbf{k}/2)+i\eta}\right]\mathbf{A}_{\mathbf{k}}^{\nu}(\omega) \tag{8}$$

where

$$\Pi_{\mathbf{k}}^{\mu} = \frac{1}{2}\sum_i(\mathbf{V}_i^{\mu}e^{-\mathbf{k}\cdot\mathbf{x}_i} + e^{+i\mathbf{k}\cdot\mathbf{x}_i}\mathbf{V}_1^{\mu}) \tag{9}$$

with

$$\mathbf{V}_i^{\mu} = \gamma^{-1}\frac{\Pi_i^{\mu}}{m}, \qquad \Pi_i^{\mu} = \mathbf{P}_i^{\mu} - \frac{e}{c}\mathbf{A}^{0\mu}(x_i) \tag{10}$$

For the argument of ω_{n0} as well as the summation over $\mathbf{P}$ in the equation,

$$\mathbf{p}=\mathbf{p}_z, \qquad \mathbf{k}=k_z, \qquad \mathbf{s}=\pm s_z \tag{11}$$

From equations (7) and (8) it is found that

$$\sigma^{\mu\nu}(\mathbf{k}\omega) = i\frac{e^2}{\omega}\left[\chi^{\mu\nu}(\mathbf{k}\omega) - \frac{N\gamma^{-1}}{m}\mathbf{T}_{\mathbf{k}}^{\mu\nu}\right] \tag{12}$$

where $\chi^{\mu\nu}(\mathbf{k}\omega)$ is the electron response tensor defined as

$$\chi^{\mu\nu}(\mathbf{k}\omega) = \sum_{np}\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \times\left[\frac{1}{\omega-\omega_{n0}(\mathbf{p},\mathbf{p}+\hbar\mathbf{k}/2)+i\eta}-\frac{1}{\omega-\omega_{n0}(\mathbf{p},\mathbf{p}-\hbar k/2)+i\eta}\right] \tag{13}$$

Since $\alpha^{\mu\nu}(\mathbf{k}\omega)$ and $\sigma^{\mu\nu}(\mathbf{k}\omega)$ are interrelated as

$$\alpha^{\mu\nu}(\mathbf{k}\omega) = i\frac{4\pi e^2}{\omega}\alpha^{\mu\nu}(\mathbf{k}\omega) \tag{14}$$

then from equations (12) and (14) it is found that

$$\alpha^{\mu\nu}(\mathbf{k}\omega) = -\frac{\omega_p^2}{\omega^2}\gamma^{-1}\mathbf{T}_{\mathbf{k}}^{\mu\nu} + \bar{\alpha}^{\mu\nu}(\mathbf{k}\omega) \tag{15}$$

where

$$\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega) = 4\pi e^2 \frac{\chi^{\mu\nu}}{\omega^2}(\mathbf{k}\omega) \tag{16}$$

The matrix elements and excitation frequencies that appear in equation (16) are those appropriate for a system of electrons with Coulomb, spin, and external magnetic field interactions.

2. TRANSVERSE SUM RULES

The complete modified polarizability tensor $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is known to be expressible in terms of corresponding "external" quantities $\hat{\bar{\alpha}}^{\mu\nu}(\mathbf{k}\omega)$ as

$$\bar{\alpha}(\mathbf{k}\omega) = \hat{\bar{\alpha}}(\mathbf{k}\omega)[\Delta - \hat{\bar{\alpha}}(\mathbf{k}\omega)]^{-1}\Delta \tag{17}$$

where

$$\begin{aligned} \Delta &= \mathbb{1} - n^2\mathbf{T} \\ n &= \frac{kc}{\omega} \\ \mathbf{T} &= \mathbb{1} - \frac{\mathbf{k}\cdot\mathbf{k}}{k^2} \end{aligned} \tag{18}$$

with

$$\mathbb{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{19}$$

$\hat{\bar{\alpha}}^{\mu\nu}(\mathbf{k}\omega)$ is known to possess a high-frequency sum-rule expansion given by

$$\hat{\bar{\sigma}}^{H'\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=1 \\ l=\text{odd}}} \frac{\hat{\bar{\Omega}}_{l+1}^{\mu\nu}(\mathbf{k})}{\omega^{l+1}} \tag{20}$$

$$\hat{\bar{\alpha}}^{H''\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=2 \\ l=\text{even}}} \frac{\hat{\bar{\Omega}}_{l+1}^{\mu\nu}(\mathbf{k})}{\omega^{l+1}} \tag{21}$$

where superscript H denotes "Hermitian part of," while prime and double prime stand for "real part of" and "imaginary part of," respectively; the $\hat{\bar{\Omega}}_{l+1}^{\mu\nu}(\mathbf{k})$ coefficients are obtained from the relation (Genga, 1988*a*,*b*, 1989, 1992*a*–*d*)

$$\hat{\hat{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k}) = \frac{4\pi e^2}{\hbar^{l-1}} \sum_{nps} \left\{ \left[\omega_{n0}\left(p, p - \frac{\hbar \mathbf{k}}{2}\right) \right]^{l-2} \langle 0|\Pi^{\mu}_{\mathbf{k}}(\tau)|n\rangle\langle n|\Pi^{\nu}_{-\mathbf{k}}(0)|0\rangle - \left[-\omega_{n0}\left(p, p + \frac{\hbar \mathbf{k}}{2}\right) \right]^{l-2} \langle 0|\Pi^{\nu}_{-\mathbf{k}}(\tau)|n\rangle\langle n|\Pi^{\mu}_{\mathbf{k}}(\tau)|0\rangle \right\}_{\tau=0} \quad (22)$$

The high-frequency expansion of $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is similar to that of $\hat{\hat{\alpha}}^{\mu\nu}(\mathbf{k}\omega)$ given by equations (20) and (21) with $\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})$ replacing the corresponding $\hat{\hat{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k})$; the relationship between the two sets of coefficients up to $l = 4$ is the same as for the nonrelativistic case.

The Hamiltonian of the system that satisfies equation (24) is given by

$$\mathbf{H} = \sum_i \frac{m}{2} \mathbf{V}_i^2 + \frac{1}{2} \sum_{\substack{i \mp j \\ ij}} \mathbf{U}(|\mathbf{x}_i - \mathbf{x}_j|)$$

$$= \sum_i \gamma^{-2} \frac{\Pi_i^2}{2m} + \frac{1}{2} \sum_{\substack{ij \\ i \mp j}} \mathbf{U}(|\mathbf{x}_i - \mathbf{x}_j|) \quad (23)$$

where $\mathbf{U}(|\mathbf{x}_i - \mathbf{x}_j|)$ is the velocity-independent interaction potential between a pair of particles.

Finally, the evaluation of the frequency moments (up to $l = 4$) is considered. It is known that in an anisotropic system, in the presence of an external magnetic field, the dielectric tensor has six independent elements; consequently, $\bar{\alpha}^{\mu\nu}$ is nondiagonal. Hence, both even and odd moments of $\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})$ exist with real diagonal and off-diagonal elements satisfying the symmetric condition

$$\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k}) = \bar{\Omega}^{\nu\mu}_{l+1}(\mathbf{k}) \quad (24)$$

and the imaginary off-diagonal elements satisfying the antisymmetric condition

$$\bar{\Omega}^{\mu\nu}_{l+1}(\mathbf{k}) = -\bar{\Omega}^{\nu\mu}_{l+1}(\mathbf{k}) \quad (25)$$

The first moment leads to

$$\hat{\hat{\Omega}}^{\mu\nu}_2(\mathbf{k}) = 4\pi e^2 \sum_{np} \left[\frac{\langle 0|\pi^{\mu}_{\mathbf{k}}(\tau)|n\rangle\langle n|\pi^{\nu}_{-\mathbf{k}}(0)|0\rangle}{\omega_{n0}(p, p + \hbar\mathbf{k}/2)} + \frac{\langle 0|\pi^{\nu}_{-\mathbf{k}}(0)|n\rangle\langle n|\pi^{\nu}_{\mathbf{k}}(\tau)|0\rangle}{\omega_{n0}(p, p - \hbar\mathbf{k}/2)} \right]_{\tau=0}$$

$$= \gamma^{-1}\omega_p^2 \mathbf{L}^{\mu\nu}_{\mathbf{k}} \quad (26)$$

where

$$\mathbf{L}^{\mu}_{\mathbf{k}} = \frac{k^{\mu}k^{\nu}}{k^2} \quad (27)$$

The second moment leads to

$$\hat{\hat{\Omega}}_3^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{\hbar} \sum_{np} [\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle - \langle 0|\Pi_{-\mathbf{k}}^{\nu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\nu}(\tau)|0\rangle]_{\tau=0}$$

$$= \frac{2\pi e^2}{\hbar} [\langle 0|[\Pi_{\mathbf{k}}^{\mu}(\tau), \Pi_{-\mathbf{k}}^{\nu}(0)] - [\Pi_{-\mathbf{k}}^{\nu}(0), \Pi_{\mathbf{k}}^{\mu}(\tau)]|0\rangle]_{\tau=0}$$

$$= i\omega_p^2 \gamma^{-2} \frac{eB_\eta^0}{mc} \varepsilon^{\mu\nu\eta} \tag{28}$$

The third moment is given by

$$\hat{\hat{\Omega}}_4^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{\hbar^2} \sum_{np} \left[\omega_{n0}\left(p, p - \frac{\hbar \mathbf{k}}{2}\right)\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \right.$$

$$\left. - \omega_{n0}\left(p, p + \frac{\hbar\mathbf{k}}{2}\right)\langle 0|\Pi_{-\mathbf{k}}^{\nu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\nu}(\tau)|0\rangle \right]_{\tau=0}$$

$$= \frac{2\pi e^2}{\hbar^2} \langle 0|[[\Pi_{\mathbf{k}}^{\mu}(\tau), H], \Pi_{-\mathbf{k}}^{\nu}(0)] + [[\Pi_{-\mathbf{k}}^{\nu}(0), H], \Pi_{\mathbf{k}}^{\mu}(\tau)]|0\rangle|_{\tau=0}$$

$$= \gamma^{-4} \frac{\omega_p^2 eB_\eta^0}{2mc} k^\alpha \langle 0|\varepsilon^{\mu\eta\nu} \frac{\partial}{\partial x^\alpha} + \varepsilon^{\mu\eta\alpha} \frac{\partial}{\partial x^\nu} + \varepsilon^{\alpha\eta\nu} \frac{\partial}{\partial x^\mu}$$

$$+ i\varepsilon^{\mu\eta\nu} \frac{eB_\eta^0}{2mc} (x^\nu - x^\mu)|0\rangle - \gamma^{-4} \frac{\omega_p^2 eB_\eta^0}{4mc} k^\mu \langle 0|\varepsilon^{\alpha\eta\nu} \frac{\partial}{\partial x^\alpha}$$

$$+ i\varepsilon^{\alpha\eta\nu} \frac{eB_\eta^0}{2mc} \chi^\nu|0\rangle - \gamma^{-4} \frac{\omega_p^2 eB_\eta^0}{4mc} k^\nu \langle 0|\varepsilon^{\mu\eta\alpha} \frac{\partial}{\partial x^\alpha} - i\varepsilon^{\mu\eta\alpha} \frac{eB_\eta^0}{2mc} \chi^\mu|0\rangle$$

$$- \gamma^{-4} \frac{\omega_p^2 eB_\eta^0}{2mc} k^\alpha k^\mu \langle 0|2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial\chi^\alpha\partial\chi^\nu} + i\varepsilon^{\alpha\eta\beta}\chi^\beta \frac{\partial}{\partial\chi^\nu}$$

$$- i\varepsilon^{\alpha\eta\nu} \frac{\partial}{\partial\chi^\alpha} - \varepsilon^{\mu\eta\alpha} \frac{eB_\eta^0}{2mc} (x^\mu)^2|0\rangle$$

$$- \gamma^{-4}\omega_p^2 \frac{eB_\eta^0}{2mc} k^\alpha k^\nu \langle 0|2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial\chi^\alpha\partial\chi^\alpha} - i\varepsilon^{\alpha\eta\eta}\chi^\alpha \frac{\partial}{\partial\chi^\alpha}$$

$$+ i\varepsilon^{\alpha\eta\beta}\chi^\beta \frac{\partial}{\partial\chi^\alpha} - \varepsilon^{\mu\eta\alpha} \frac{eB_\eta^0}{2mc} (x^\mu)^2|0\rangle$$

$$- \gamma^{-4}\omega_p^2 \frac{eB_\eta^0}{2mc} k^\alpha k^\nu \langle 0|2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial\chi^\alpha\partial\chi^\alpha} - i\varepsilon^{\alpha\eta\mu}\chi^\alpha \frac{\partial}{\partial\chi^\mu}$$

$$- i\varepsilon^{\alpha\eta\mu}\chi^\alpha \frac{\partial}{\partial\chi^\nu} + [\varepsilon^{\mu\eta\alpha}(\chi^\alpha)^2\delta^{\mu\nu} - \varepsilon^{\mu\eta\nu}\chi^\mu\chi^\nu] \frac{eB_\eta^0}{2mc} |0\rangle$$

$$+ \gamma^{-2}\omega_p^4 \langle 0|\mathbf{L}_{\mathbf{k}}^{\mu\nu} + \frac{1}{N} \sum_{\mathbf{q}} \mathbf{L}_{\mathbf{q}}^{\mu\nu}(S_{\mathbf{k}-\mathbf{q}} - S_{\mathbf{k}})|0\rangle \tag{29}$$

The fourth moment leads to

$$\begin{aligned}
\hat{\hat{\Omega}}_5^{\mu\eta}(\mathbf{k}) &= \frac{4\pi e^2}{\hbar^3}\sum_{np}\left\{\left[\omega_{n0}\left(p, p-\frac{\hbar k}{2}\right)\right]^2 \langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle\right.\\
&\quad\left.-\left[-\omega_{n0}\left(p, p+\frac{\hbar k}{2}\right)\right]^2\langle 0|\Pi_{\mathbf{k}}^{\nu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\mu}(\tau)|0\rangle\right\}_{\tau=0}\\
&= \frac{2\pi e^2}{\hbar^3}\langle 0|[[[\Pi_{\mathbf{k}}^{\mu}, \mathbf{H}], \mathbf{H}], \Pi_{-\mathbf{k}}^{\nu}(0)] - [[[\Pi_{-\mathbf{k}}^{\nu}(0), \mathbf{H}], \mathbf{H}], \Pi_{-\mathbf{k}}^{\mu}(\tau)]|0\rangle|_{\tau=0}\\
&= -\gamma^{-6}\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha\langle 0|\varepsilon^{\mu\eta\nu}\delta^{\mu\nu}\frac{(eB_\eta^0)^2}{4m^2c^2}\chi^\mu\\
&\quad+\frac{7}{4}\varepsilon^{\mu\eta\nu}\frac{eB_\eta^0}{mc}\chi^\mu\frac{\partial}{\partial\chi^\nu}+\varepsilon^{\nu\eta\alpha}\frac{(eB_\eta^0)^2}{8m^2c^2}(x^\nu)^2\frac{\partial}{\partial\chi^\mu}\\
&\quad+\varepsilon^{\mu\eta\alpha}\frac{(eB_\eta^0)^2}{8m^2c^2}(\chi^\mu)^2\frac{\partial}{\partial\chi^\nu}-i\varepsilon^{\mu\nu\eta}\frac{(eB_\eta^0)^3}{16m^3c^3}(\chi^\mu)^3\\
&\quad+i6\varepsilon^{\mu\eta a}\delta^{\mu\nu}\frac{eB_\eta^0}{mc}\frac{\partial}{\partial\chi^\alpha}|0\rangle-\gamma^{-6}\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha k^\mu\langle 0|i\frac{7}{4}\varepsilon^{\mu\alpha\nu}\frac{\partial^2}{\partial\chi^\alpha\partial\chi^\alpha}\\
&\quad+\frac{17}{8}\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}\chi^\nu\frac{\partial}{\partial\chi^\alpha}+i\frac{7}{4}\varepsilon^{\nu\eta\alpha}\frac{(eB_\eta^0)^2}{m^2c^2}(x^\nu)^2|0\rangle\\
&\quad-\gamma^{-6}\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha k^\nu\langle 0|i\frac{7}{4}\varepsilon^{\mu\eta\alpha}\frac{\partial^2}{\partial\chi^\alpha\partial\chi^\alpha}+\frac{17}{8}\varepsilon^{\mu\eta\alpha}\frac{eB_\eta^0}{mc}\chi^\mu\frac{\partial}{\partial\chi^\alpha}\\
&\quad+i\frac{7}{4}\varepsilon^{\mu\eta\alpha}\frac{(eB_\eta^0)^2}{m^2c^2}(x^\mu)^2|0\rangle-\gamma^{-6}\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha k^\alpha\langle 0|i6\varepsilon^{\mu\eta\alpha}\frac{\partial^2}{\partial\chi^\alpha\partial\chi^\alpha}\\
&\quad+i6\varepsilon^{\alpha\eta\nu}\frac{\partial^2}{\partial\chi^\alpha\partial\chi^\mu}+i\frac{3}{2}\varepsilon^{\mu\eta\nu}\frac{\partial^2}{\partial\chi^\alpha\partial\chi^\alpha}+\frac{3}{2}\varepsilon^{\mu\eta\alpha}\delta^{\mu\nu}\frac{eB_\eta^0}{mc}\\
&\quad+3\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}\chi^\nu\frac{\partial}{\partial\chi^\nu}+3\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}\chi^\nu\frac{\partial}{\partial\chi^\mu}-i\frac{15}{4}\varepsilon^{\mu\eta\nu}\frac{(eB_\eta^0)}{m^2c^2}(\chi^\mu)^2\\
&\quad+i\varepsilon^{\mu\eta\nu}(\chi^\nu-\chi^\mu)\frac{eB_\eta^0}{mc}|0\rangle+i\gamma^{-4}\omega_p^4\frac{eB_\eta^0}{2mc}\langle 0|\mathbf{L}_k^{\mu\nu}\\
&\quad+\frac{1}{N}\sum_q(\varepsilon^{\mu\nu\alpha}\mathbf{L}_q^{\alpha\nu}+\varepsilon^{\alpha\eta\nu}\mathbf{L}_q^{\alpha\mu})(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{k}})|0\rangle
\end{aligned}\tag{30}$$

I choose the k system in which

$$\begin{aligned}
\mathbf{k} &= (0, 0, k)\\
\mathbf{B}^0 &= (B_x^0, 0, B_z^0)
\end{aligned}\tag{31}$$

and

$$\begin{aligned} q_x &= q \sin\theta \cos\theta \\ q_y &= q \sin\theta \sin\theta \\ q_z &= q \cos\theta \end{aligned} \tag{32}$$

to obtain an explicit expression $\hat{\hat{\Omega}}^{\mu\nu}_{l+1}(\mathbf{k})$. Further, the Landau gauge

$$A^0 = \frac{1}{2}(0, B_z^0 x - ZB_x^0, 0) \tag{33}$$

is used to obtain components of the external magnetic field given by equation (31). The wave function $|0\rangle$ is known (Johnson and Lipmann, 1949; Genga, 1992*d*) to be given by

$$|0\rangle = (2\pi)^{1/2}\lambda^{-1}\, e^{(y-y_0)^2/4\lambda} + iP_z/\hbar \tag{34}$$

where

$$\begin{aligned} \lambda &= \frac{\hbar}{m\Omega} \\ y_0 &= -\frac{2c}{e} P_y \\ y &= B_z^0 x - B_x^0 z = \frac{c}{e} P_y \end{aligned} \tag{35}$$

and $\Omega = -eB^0/mc$ is the electron cyclotron frequency.

3. LONG-WAVELENGTH LIMIT

In the long-wavelength limit ($k \to 0$) equations (26)–(32) yield

$$\begin{aligned}
&\hat{\hat{\Omega}}_2^{11}(\mathbf{k}) = \hat{\hat{\Omega}}_2^{22}(\mathbf{k}) = 0 \\
&\hat{\hat{\Omega}}_2^{33}(\mathbf{k}) = \gamma^{-1}\omega_p^2 \\
&\hat{\hat{\Omega}}_3^{12}(\mathbf{k}) = -\bar{\Omega}_3^{21}(\mathbf{k}) = i\gamma^{-2}\omega_p^2\Omega\cos\theta \\
&\hat{\hat{\Omega}}_3^{23}(\mathbf{k}) = -\hat{\hat{\Omega}}_3^{32}(\mathbf{k}) = i\gamma^{-2}\omega_p^2\Omega\sin\theta \\
&\hat{\hat{\Omega}}_4^{11}(\mathbf{k}) = \hat{\hat{\Omega}}_4^{22}(\mathbf{k}) = -2\gamma^{-2}\omega_p^2 E_{\text{corr}} k^2 \\
&\hat{\hat{\Omega}}_4^{13}(\mathbf{k}) = \hat{\hat{\Omega}}_4^{31}(\mathbf{k}) = 0 \\
&\hat{\hat{\Omega}}_4^{33}(\mathbf{k}) = \gamma^{-2}\omega_p^4 - \gamma^{-4}\frac{\omega_p^2}{m}\left(\frac{3(P_{\mathrm{F}}^{(0)})^2}{m} - \frac{4}{15}\gamma^2 E_{\text{corr}}\right)k^2 \\
&\hat{\hat{\Omega}}_5^{12}(\mathbf{k}) = -\hat{\hat{\Omega}}_5^{21}(\mathbf{k}) = i\gamma^{-6}\frac{\omega_p^2\Omega}{8m}\left(\frac{3(P_{\mathrm{F}}^{(0)})^2}{m} + \frac{16}{15}\gamma^2 E_{\text{corr}}\right)k^2\cos\theta \\
&\hat{\hat{\Omega}}_5^{23}(\mathbf{k}) = -\hat{\hat{\Omega}}_5^{32}(\mathbf{k}) = -i\gamma^{-6}\frac{\omega_p^2\Omega}{8m}\left(\frac{(15P_{\mathrm{F}}^{(0)})^2}{m} - \frac{24}{15}\gamma^2 E_{\text{corr}}\right)k^2\sin\theta
\end{aligned} \tag{36}$$

where $P_F^{(0)}$ is the lowest Landau level Fermi momentum and E_{corr} is the (negative) correlation energy per particle.

4. SPIN EFFECT

The spin effect on the undamped high-frequency, quasi-one-dimensional quantum plasma waves in an external magnetic field is considered in this section by using the high-frequency sum rules. The high-frequency modes of interest are the "ordinary" and "extraordinary" modes with cutoff frequency $\omega_2 = \frac{1}{2}\Omega[1 + (1 + 4\omega_p^2/\Omega^2)^{1/2}]$; all the modes propagating along and across the external magnetic field are considered in a coordinate system where $\mathbf{k} = (0, 0, k)$ and $\mathbf{B}^0$ is in the $\mathbf{x}-\mathbf{y}$ plane; i.e., the k system.

4.1. Ordinary Mode

It is known (Genga, 1988*b*, 1989, 1992*c*–*d*) that for propagation along and across the external magnetic field the ordinary mode does not exist; however, for parallel propagation, the longitudinal mode oscillating at the frequency exists instead. When a small perturbation is applied to the dispersion relation the plasmon frequency of the form

$$\omega^2(k) = \gamma^{-1}\omega_p^2\left[1 - \gamma^{-5/2}\frac{\omega_p^2}{m}\left(6E_F - \frac{4}{15}\gamma^2 E_{corr}\right)k^2\right] \tag{37}$$

is found, where

$$E_F = \frac{(P_F^{(0)})^2}{2m} \tag{38}$$

is the Fermi energy per particle corresponding to the lowest Landau level. From equation (37) it can be seen that the spin term either enhances or reduces the frequency of oscillation, depending on the orientation of the spin to the external magnetic field, unlike in the nonrelativistic quantum plasma case (Genga, 1992*c*), where it is found that the spin effect does not exist for such waves; this can also be confirmed by setting $\gamma = 1$ in equation (37). Further, by setting $s = 0$ in equation (37), the result of the relativistic quantum plasmas without spins (Genga, 1992*d*) is recovered.

4.2. Extraordinary Mode

In this case it is found that an application of a small perturbation to the dispersion relation leads to a frequency of oscillation of the form

$$\omega^2(\mathbf{k}) = \gamma^{-1}\omega_2^2\left[1 + \frac{2}{3}\gamma^{1/2}\left(\frac{c_2^2}{\omega_2^2} + 2\frac{\gamma^{-2}\omega_p^2}{15m\omega_2^4}E_{corr}\right)k^2\right] \tag{39}$$

for propagation along the external magnetic field, and

$$\omega^2 = \gamma^{-1}\omega_2^2\left\{1 + \gamma^{1/2}\left[\frac{c^2}{\omega_p^2} - \frac{2\gamma^{-4}}{m\omega_2^2}\left(3E_F - \frac{1}{15}\gamma^2 E_{corr}\right)\right]k^2\right\} \tag{40}$$

for propagation across the external field, i.e., $\theta = 90°$. From equations (39) and (40) it can be seen as in equation (37) that the spin term either enhances or reduces the frequency of oscillation, depending on the orientation of the spin to the external magnetic field, unlike in the nonrelativistic quantum plasma case (Genga, 1992*c*); the results of the nonrelativistic case can be obtained from equations (39) and (40) by setting $\gamma = 1$. Further, by setting $s = 0$ the results for relativistic quantum plasmas without spins (Genga, 1992*d*) are recovered.

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